RESEARCH DEPARTMENT

An analogue multiplier for correlation

TECHNOLOGICAL REPORT No. PH-12

UDC 621.382 1967/46

THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION

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Technological Report No. PH-12
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This Report uses SI units in accordance with B.S. document PD 5686.

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October 1967

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SUMMARY

This report describes an analogue multiplier using a variable area method in which a repetitive waveform is produced with a mark-to-space ratio dependent upon one multiplicand and an amplitude dependent upon the other. The resulting waveform is integrated by a resistance-capacity network and the d.c. voltage so produced is fed to a diode chopper which is switched by a 1·5 kHz square-wave. The 1·5 kHz waveform is amplified to operate a peak level Bruel and Kjaer chart recorder.

1. INTRODUCTION

An analogue correlator has been developed for sound insulation and acoustic tests in studios or other enclosures.[1] Correlation techniques allow the separation of signals which have arrived by paths of different transit times or originated from different sources. The cross-correlation function $F(\tau)$ of two signals $f_1(t)$ and $f_2(t)$ is given by

$$F(\tau) = \overline{f_1(t)\ f_2(t-\tau)}$$

$$= \overline{f_1(t-\tau)\ f_2(t)}$$

where the bar denotes an average taken over all time t. It is in fact the average product of the instantaneous value of one signal undelayed and that of the other signal delayed by a time, τ. In practice, averaging of the product over a few seconds is a close enough approximation to the ideal. To derive the cross-correlation function a multiplier and integrator are required.

The first multiplier to be used was a "quarter-squaring" multiplier using valves. It was susceptible to drift caused by changes in temperature of the valve cathodes and variations of the mains supply, and therefore had to be re-balanced frequently while the equipment was in use. The signal-to-noise ratio was limited to approximately 25 dB.

The possibility of using a Hall-effect multiplier was also investigated but this was rejected because of the inherent low efficiency and the difficulty of avoiding crosstalk.

In view of the shortcomings of both the "quarter-squaring" and Hall-effect multipliers a transistor analogue multiplier based on a circuit by D.L.A. Barber[2] was investigated. A minimum output range of 35 dB was required, with a bandwidth of 8 kHz. Briefly, the circuit forms a train of rectangular pulses in which the amplitude is proportional to one variable and the mark-to-space ratio is a function of the other variable. The resultant waveform is then integrated by a simple resistance-capacity network to produce a d.c. voltage proportional to the product of the two variables.

An additional unit, a d.c. to a.c. converter, was designed to convert the d.c. output of the multiplier to a 1·5 kHz square wave to drive a peak-level Bruel and Kjaer recorder. The performance of this converter proved to be the biggest single factor in determining the signal-to-noise ratio obtainable from the apparatus, and much of the time of development of the whole equipment was devoted to the perfection of the diode chopper which was finally adopted.

2. MULTIPLIER

2.1. Principle of Operation

Barber's original circuit[2] was intended to take separate positive and negative signals in each of the two channels X and Y. Since the present apparatus had to accept alternating signals with both polarities the positive and negative sides were ganged together and fed by a common Y input to the multiplier. The modifications also included the substitution of transistors with higher cut-off frequency to improve the shapes of the rectangular pulses formed by the circuit.

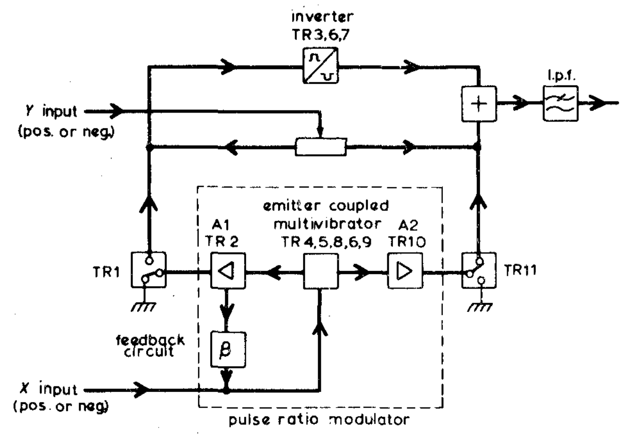

Fig. 1 - Block diagram of multiplier (Transistor numbers refer to Fig. 4)

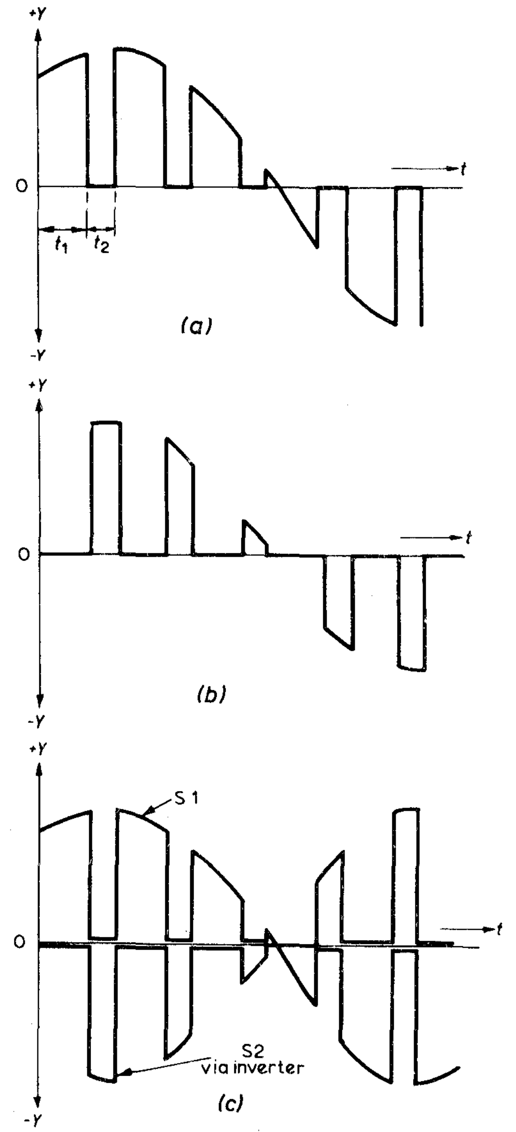

Fig. 2 - Production of waveform with area proportional to product of two inputs
(*a*) Waveform at TR1 (*b*) Waveform at TR2
(c) Waveform at adder

Fig. 1 is a block diagram of the multiplier, which is capable of generating the product of both positive and negative functions. The pulse-ratio modulator which is discussed in Section 2.2, formed by the emitter-coupled multivibrator and the two amplifiers A1 and A2, generates rectangular waveforms with a nominal frequency of 100 kHz and a mark-to-space ratio depending on the input variable *X*. The two outputs from the modulator, which are in antiphase, drive the bases of the two switches.

These two switches periodically short-circuit the *Y* input and leave an output waveform as shown in Figs. 2(*a*) and 2(*b*). The output from TR1 is reversed in sign by the inverter stage and added to the output of TR11 to form the resultant waveform at the adder as shown in Fig. 2(c). This output is then integrated by a resistance-capacity network giving a final output which, as shown below, is the product of the input variables *X* and *Y* and a constant K.

The integrating circuit was arranged to give five switched time constants of ½, 1, 2, 4 and 7 seconds within the limits of 10% and 90% of the capacitor charge.

2.2. Pulse-Ratio Modulation

Pulse-ratio modulation[3],[4] is a combination of pulse-frequency modulation and pulse-width modulation, in which the pulse width and pulse frequency are varied simultaneously according to the input signal.

The emitter-coupled multivibrator[5] produces a rectangular waveform governed by one timing element and having a mark-to-space ratio dependent upon two emitter currents. The emitter currents of the multivibrator are the collector currents of two transistors in a long-tailed pair which are controlled by the input voltage* to the base of one transistor of the long-tailed pair.

Fig. 3(*a*) shows a simplified diagram of the long-tailed pair, while Fig. 3(*b*) is an idealized graph of the relationship between the currents i_1 and i_2 in the emitters of the multivibrator transistors and the input *X*. To improve the linearity of this relationship a negative feed-back signal derived from the collector load of amplifier A1 is fed back to the *X* input. The complete circuit diagram is shown in Fig. 4.

* For simplicity in the discussion which follows (Sections 2-3, Figs. 3 and 5) the input voltage is normalized as its ratio to the maximum input voltage i.e. $X \leqslant 1$.

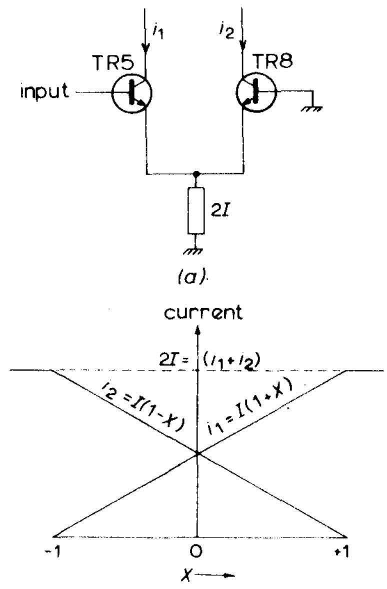

Fig. 3 - Time-division by long-tailed pair and multivibrator

(a) Long-tailed pair

(b) Idealized graph of collector currents i_1 and i_2

(Transistor numbers refer to Fig. 4)

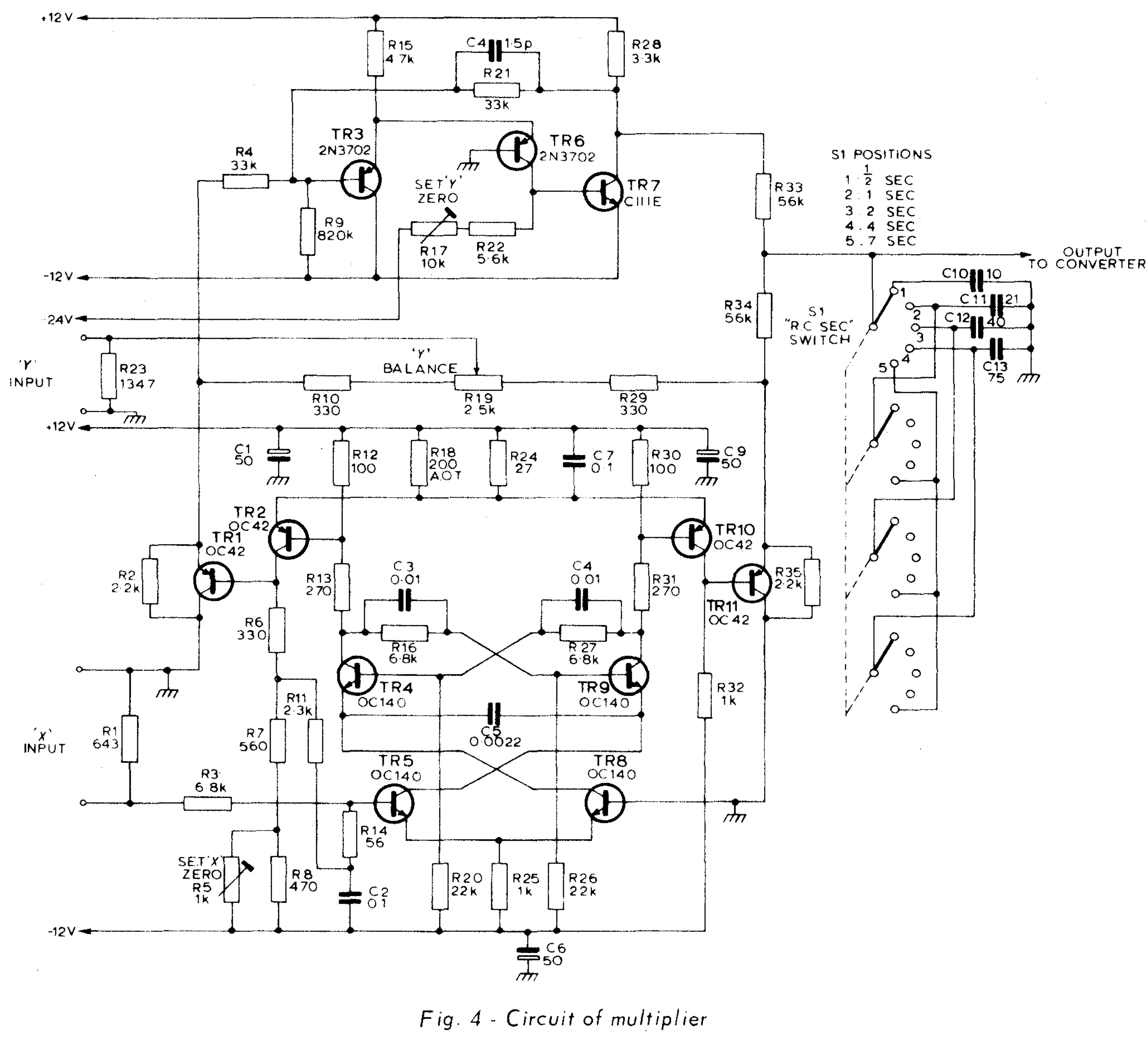

Fig. 4 - Circuit of multiplier

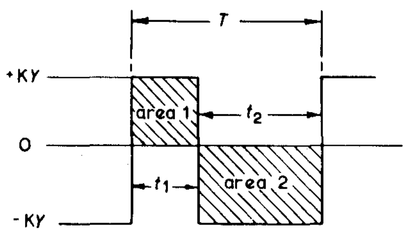

Fig. 5 - Derivation of product by integration of waveform

2.3. Derivation of the Product

Now from Fig. 3(*b*), $i_1 = I\,(1 + X)$ and $i_2 = I\,(1 - X)$ where $2I$ is the total collector current from the two transistors. Moreover the times t_1, t_2 occupied by the multivibrator in the two states are inversely proportional to the emitter currents of the two transistors. It follows that

$$\frac{t_1}{t_2} = \frac{i_2}{i_1} = \frac{1 - X}{1 + X} \qquad (1)$$

so that

$$X = \frac{t_2 - t_1}{t_2 + t_1} \qquad (2)$$

The voltage waveform at the input to the low-pass filter is as shown in Fig. 5, in which K is a constant depending on the constants of the circuit. The d.c. component $\overline{V}$ of this voltage is given by

$$\overline{V} = \frac{KY(t_1 - t_2)}{t_1 + t_2} = -KXY \qquad (3)$$

The period $(t_1 + t_2)$ of the multivibrator is proportional to the sum of the reciprocals of the emitter currents so that

$$t_1 + t_2 = \tfrac{1}{2} t_0 \left(\frac{1}{1 - X} + \frac{1}{1 + X}\right)$$

$$= \frac{t_0}{1 - X^2} \qquad (4)$$

where t_0 is a constant.

This feature of the system, which was devised by Schaefer, enables it to deal with a very much larger range of X than one in which the frequency remains constant and only the mark-to-space ratio is changed, since it lengthens the "on" pulses for high X inputs. It therefore increases the ratio of useful signal to unwanted switching transients and for the maximum input of X the improvement in signal-to-noise ratio obtainable is approximately 11 dB. Since the demodulation is effected afterwards by a simple low-pass filter the variation in carrier frequency introduces no additional filtering problems.

2.4. Effects of Switching Delays

The collector currents of the multivibrator transistors are determined by the tail resistor of the long-tailed pair and the resistor is chosen to avoid "bottoming" these collectors. Saturation is prevented and fast switching results. However, saturation may occur in the amplifiers TR2 and TR10 and the output switches, caused by high input signals, and thus delay the switching of these transistors. An output switch cannot open until its own stored charge is removed and will not close until the amplifier stored charge is extracted. These delays in switching "on" and "off" may be made equal by adjusting the emitter bias voltage of these transistors, thus changing the base drive to the amplifiers. When the amplifier is driven hard on, it quickly switches the output transistor, but excess charge is accumulated in its own base.

If both output switches are to switch together at all mark-to-space ratios the "on" and "off" switching delays must be made equal by the adjustment of R18, Fig. 4. If the delays of the two switches are not equal an offset voltage is developed at the output of the multiplier in the zero output condition when one or both inputs are not connected. Adjustment of R18 is required only if one of the output switches has to be replaced.

2.5. Linearity

Fig. 6 shows the integrated d.c. voltage at the output as a function of the actual X input voltage with constant values of Y maintained as $+10$ and -10 volts respectively. The linearity is within $\pm$ 0·3% of full output.

2.6. Effect of Phase Errors in the X Input

The effect of a phase lag ϕ in the X circuits will be to cause the product to have the value $XY\cos\phi$. A phase lag of 16° initially present in the X-drive circuit was corrected by modification of the feed-back loop from the collector of TR2 to the base of TR5, and the final result was tested by applying constant X and Y input amplitudes and varying the phase of the X input with respect to the Y input. This was done by means of equipment giving delays variable in steps of 10 μs up to 100 μs. By working at 5 kHz and 10 kHz, therefore, phase delays in steps of 18° and 36° respectively were obtained. Fig. 7 shows the experimental relationship between output and phase lag (curve (*a*)) compared with the theoretical law (curve (*b*)). The phase is seen to be within 2° of the correct value which would cause negligible error in practice. The results at 5 kHz and 10 kHz agreed with each other to within 1·3% of maximum output.

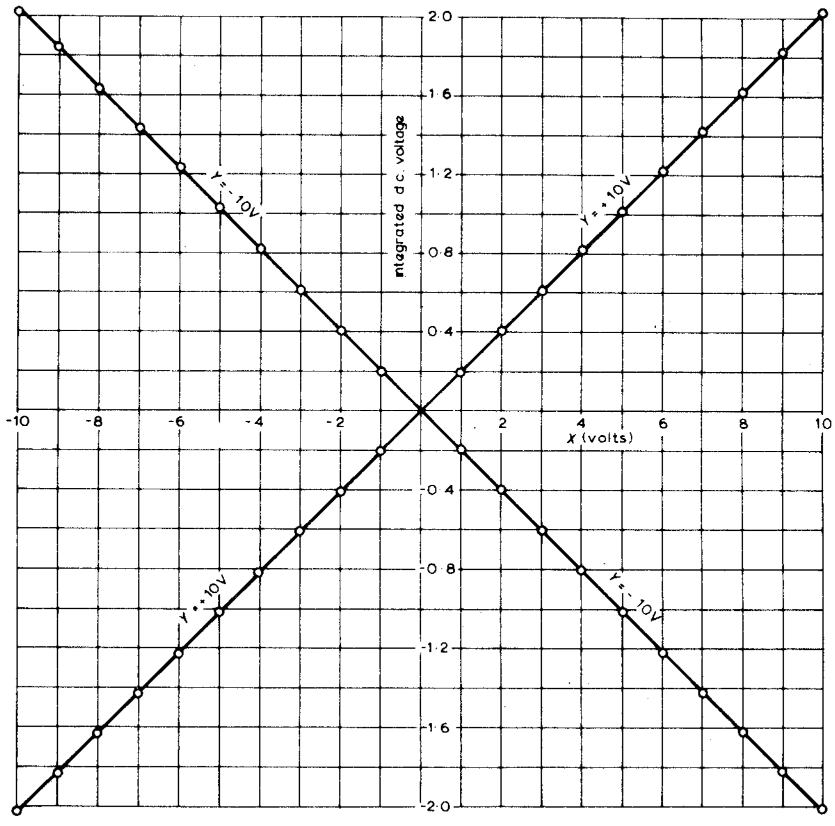

Fig. 6 - Relationship between integrated output and X input voltage for constant positive and negative Y-inputs

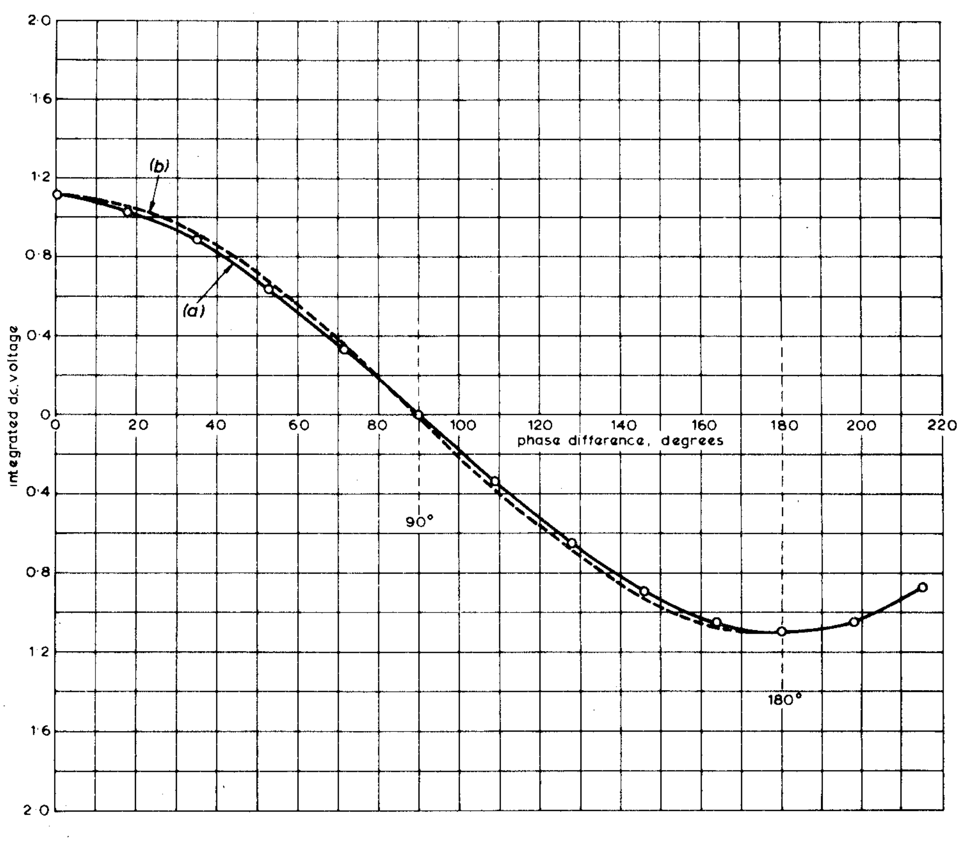

Fig. 7 - Phase measurement at 5 kHz and 10 kHz using 50 μs and 100 μs fixed delays

(a) Measured relationship between output and phase (b) Theoretical relationship (cosine function)

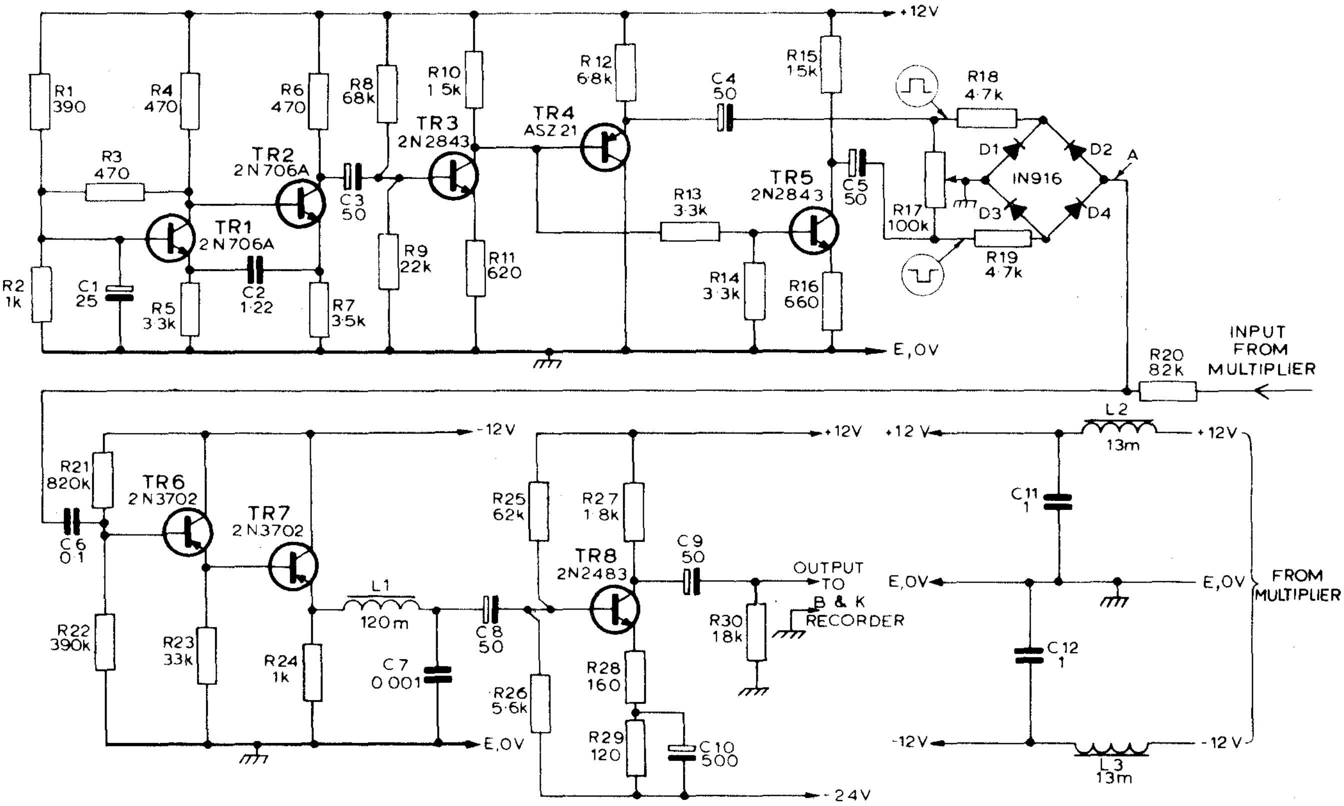

Fig. 8 - D.C. to A.C. converter

3. D.C. TO A.C. CONVERSION

Three general methods were considered. For many purposes simple mechanical inverters are used, but their life is limited by fatigue leading to contact bounce. Transistor choppers provide simple d.c. to a.c. conversion but generate 'spikes' caused by the carrier storage and interelectrode capacitance of the transistor.

It was therefore decided to build a diode chopper which produces smaller spikes but may drift more than a transistor chopper.

The basis of this circuit is a ring of four diodes shown as D1 - D4 in Fig. 8. When trains of positive and negative 1·5 kHz square waves respectively are fed to either side of the bridge as shown,

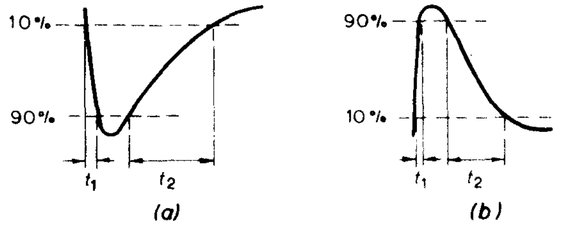

Fig. 9 - Characteristics of spikes
(a) Negative spikes (b) Positive spikes

point A, the junction of D2 and D4, is periodically earthed. The d.c. potential applied to A through R20 therefore gives rise to a 1·5 kHz square wave at A, and the peak-to-peak amplitude of this is equal to the value of the d.c. voltage applied.

It has to be ensured that the pulse shapes of the two trains of positive and negative square waves when applied to the bridge are of similar shape to ensure that asymmetry will not occur at the output.

The diode bridge is driven by an emitter-coupled multivibrator TR1 and TR2, which delivers a 1·5 kHz square wave of 2 volts peak-to-peak amplitude. This is amplified by TR3, and then phase-split by a p.n.p. and n.p.n. combination TR4 and TR5. A positive and negative train of square waves of 4 volts peak-to-peak is thereby delivered to either side of the bridge, exceeding the peak d.c. input of 2 volts as required.

Table 1 shows the amplitudes and widths of the spikes produced by the chopper. All amplitudes are in volts while times of rise and fall of the spikes are measured between 10% and 90% of peak amplitude. (See Fig. 9). Figures are given both for the output of the shopper stage itself and for the output of the subsequent amplifying stages terminated by an 18 kΩ load. The bottom section of the table represents the final result after the insertion of a

TABLE 1

Amplitudes and Widths of Spikes

	Original Pulse Polarity	Chopper Stage Output: Amplitude V	Chopper Stage Output: t_1 (μs)	Chopper Stage Output: t_2 (μs)	Converter Output: Amplitude V	Converter Output: Signal-to-Noise Ratio, dB
Diode Chopper	Positive	0·12	0·65	2·7	1·2	38
Emitter-Coupled Multivibrator	Negative	0·11	0·05	0·14	0·52	
Final Converter Circuit	Positive	–	–	–	0·05	47
With L.P. Filter	Negative	–	–	–	0·01	

low-pass filter, which improves the signal-to-noise ratio* to 47 dB.

The overall drift is within 0·3% of full output of which the chopper contributes only 0·1%.

4. TEST RESULTS AND PERFORMANCE DATA

Maximum Inputs

D.C. Y Input:	±20 volts
D.C. X Input:	±11·5 volts
Sinusoidal Inputs:	Y input + 25 dB rel. to 0·775 volts
	X input + 22 dB rel. to 0·775 volts
White Noise:	Y input, X input + 22 dB rel. to 0·775 volts
Linearity:	Within ±0·3% of maximum output (see Fig. 6)
Phase:	Within 2° at 5 kHz and 10 kHz (see Fig. 7)
Frequency Response:	±0·1 dB from 10 Hz to 10 kHz
Integration Time Constants:	½, 1, 2, 4 or 7 sec.

Drift

Overall:	40mV peak-to-peak (0·3% of full-scale output)
Chopper Stage:	2mV
Converter Output:	20mV
Range of Output:	(using B. and K. type 2304 peak-level chart recorder) 47 dB. About 3 dB more can be obtained in most operating conditions by the use of higher input.

5. CONCLUSIONS

(1) A successful analogue multiplier has been developed in which one multiplicand varies the amplitude of a train of rectangular pulses and the other the width and frequency. The maximum output working range of the analogue multiplier when used in conjunction with the Bruel and Kjaer peak-level chart recorder is 50 dB. This output range is mainly determined by the spike levels produced at the chopper.

(2) Correlation measurements using the multiplier were conducted over a period of two months and it was found that the overall drift did not exceed 40mV peak-to-peak as measured on an oscilloscope, only occasional adjustment being found necessary. It was not possible to assess accurately the effect of multiplier drift on the correlation measurements because of the variables in the chain making up the correlation equipment.

(3) The performance figures are based on only one working model and it is felt that improvements could be made in stability and output range; its present performance is fulfilling all present requirements for correlation measurements.

* This term here means the ratio of the maximum output to the maximum spurious output due mainly to the effect of spikes.

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APPENDIX

SETTING-UP AND ADJUSTMENT

A. General Setting-up

For the setting-up procedure a double-beam oscilloscope and a −10 volt d.c. supply is required.

1. Connect the converter to an oscilloscope on the 0·05 volt/cm range. Earth the output of the multiplier at the junction of R33 and R34 (Fig. 4) and adjust the 100k potentiometer across the diode bridge for a zero offset as displayed on the oscilloscope.

2. Connect a double-beam oscilloscope with two probes to either side of the adder, i.e. at the collector of TR7 and the emitter of TR11. Apply −10 volts d.c. to the *Y* input and adjust the 'Y balance' potentiometer for equal pulse amplitude.

3. Re-connect the oscilloscope on the 0·05 volt/cm range to the output of the converter. With both inputs disconnected adjust 'set 'Y' zero' potentiometer for zero offset as displayed on the oscilloscope.

4. Apply −10 volts d.c. to the *Y* input only and adjust the 'set 'X' zero' potentiometer for zero offset.

5. Repeat steps 3 and 4.

Once the multiplier has been set up the only periodic adjustment found necessary is to check the balance of the 'X' and 'Y' zero potentiometers.

B. Adjustment of Switching Delays

A further setting-up procedure as described below may be necessary if one of the output switches have to be replaced. This is to equalize the switching delays of the two output switches by the adjustment of R18.

1. Connect a voltmeter between the collector and emitter of the output transistor of the phase inverter TR7. Adjust 'set 'Y' zero' so that the potential across the transistor is 12 volts.

2. Repeat step 2 in the setting-up procedure, above.

3. Adjust the 'set 'X' zero' potentiometer for unity mark-to-space ratio at the two output switches TR1 and TR11.

4. Remove the −10 volts d.c. from the *Y* input. Connect a 1k potentiometer in place of R18.

5. Re-connect the oscilloscope to the output of the converter and adjust the 1k potentiometer for zero offset at the output.

6. Measure the resistance of the potentiometer and connect the nearest preferred value resistor for R18.

7. Re-adjust the 'X' and 'Y' zero potentiometers as explained in steps 3, 4 and 5 in the setting-up procedure above.

Re-measure the linearity of the multiplier by applying −10 volts d.c. to the *Y* input and a varying d.c. voltage from 0 to 10 volts to the *X* input, positive and negative, and noting the integrated d.c. voltage out from the multiplier.

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Printed by BBC Research Department, Kingswood Warren, Tadworth, Surrey